A comparison of the performance of DEJMPS and the $3 \to 1$ protocol at various gate and channel noise levels

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Abstract

Quantum entanglement purification is one of the key components of quantum communication with many different implementations thereof. Whilst there is an argument to be made for the application of each purification protocol, it is important to know what the differences between them are and what the advantages or disadvantages are between them. In this paper, two quantum entanglement purification protocols, DEJMPS and $3 \to 1$, will be compared to each other with regards to success rate and fidelity. The findings that will be presented showcase that the $3 \to 1$ protocol has a lower rate and a slightly higher fidelity compared to the DEJMPS for the same values of channel and gate fidelity, but taking the cost of the needed initial qubits, there is still an argument to be made for the use of the DEJMPS protocol. Furthermore, it will be shown that the gate fidelity plays a more significant role in the impact on the success rate and the fidelity compared to the impact of the fidelity of the initial pairs.

1 Introduction

Without quantum entanglement, many useful quantum communication protocols would simply not work. Therefore quantum entanglement is of the utmost importance. Many quantum communication protocols rely on maximally entangled pairs, but it is difficult to establish the maximally entangled pairs perfectly. Due to the existence of noise in the communication channels, states shared over these communication channels are partially entangled pairs instead of the desired maximally entangled pairs. In order to "fix" these partially entangled pairs, entanglement purification [6] can be performed in the form of distillation protocols. Therefore distillation protocols are of the utmost importance to quantum communication, and applications thereof like quantum teleportation [1], quantum key distribution [5] and super-dense coding [2].

As this paper is an investigation of the performance of distillation protocols in the presence of noise, there are many interesting questions to be asked regarding the distillation protocols in question. However, as for this project the DEJMPS [4] and the $3 \to 1$ [3] protocol were implemented, the research questions will be limited to these protocols.

1.1 The DEJMPS distillation protocol

The DEJMPS distillation protocol is a distillation protocol that performs $2 \to 1$ distillation. The state of the pairs that distillation is performed on, must of the Werner form:

$$\rho_{AB}(p) = p\,|\phi_{00}\rangle\,\langle\phi_{00}| + \frac{1-p}{4} I_4$$

Then to perform the DEJMPS distillation protocol Alice will perform a unitary operation U_A on her qubits defined as:

$$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle), \quad |1\rangle \to \frac{1}{\sqrt{2}}(|1\rangle - i\,|0\rangle)$$

Bob performs a similar, yet different unitary operation U_B:

$$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle), \quad |1\rangle \to \frac{1}{\sqrt{2}}(|1\rangle + i\,|0\rangle)$$

After this, Alice and Bob apply a CNOT on their respective qubits and measure the target qubit. If the outcomes are the same, they keep the control pair.

1.2 The $3 \to 1$ distillation protocol

The $3 \to 1$ distillation protocol is, as the name quite literally illustrates, a distillation protocol that performs $3 \to 1$ distillation. How the protocol does this, however, is somewhat more complicated than that. The protocol does this $3 \to 1$ distillation via two bilateral CNOT operations. In the first step of the protocol depolarization is applied to the three identical copies of a given state:

$$\frac{1-F}{3}(|\phi_{01}\rangle\,\langle\phi_{01}| + |\phi_{10}\rangle\,\langle\phi_{10}| + |\phi_{11}\rangle\,\langle\phi_{11}|) + F\,|\phi_{00}\rangle\,\langle\phi_{00}|$$

After this, the following step can commence. In this second step, CNOT operations are applied bilaterally for the source pair A_3, B_3 and the target pair A_2, B_2 first and then followed by CNOT operations bilaterally for the for the source pair A_1, B_1 and the target pair A_3, B_3. This can be seen more clearly in figure 1: Then, the two-qubit states A_1, A_2 and B_1, B_2 can be measured locally in the Bell basis. If the measurements are the same, the state is kept of pair A_3, B_3, otherwise, it is discarded. To improve fidelity even further, these three steps can be repeated by using the output states of the last step as the input states for the next round until the fidelity becomes greater than the desired fidelity.

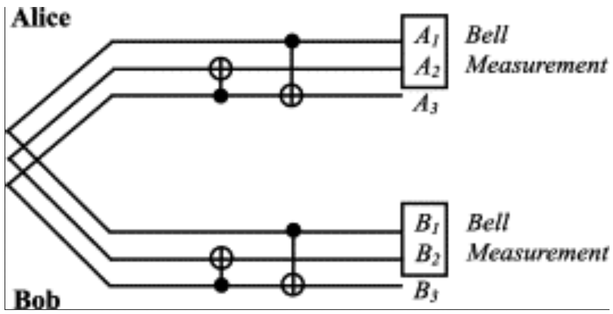

Figure 1: The $3 \rightarrow 1$ distillation protocol, figure from [3]

2 Research questions

In this paper it will be explored which of the two protocols described above perform better in a broad range of possible network setups. To do this the following research question will be answered: Which protocol is better at achieving high fidelity quantum states at what levels of channel- and gate noise?

This question will be answered using two sub-questions to explore the impacts of the noise values on separate types of results:

- For varying levels of channel- and gate fidelity, what is the probability of the protocol succeeding?
- For varying levels of channel- and gate fidelity, what is the fidelity of the resulting state when the protocol succeeds?

Answering these two questions will generate insights into which protocol is better suited for situations with various types of noise. It can also help inform on how well suited the protocols are for certain situations.

3 Methodology

To answer the research questions, the netqasm package to simulate both protocols for various levels of noise on both the gates and the channel. In netqasm values for the fidelity of both gates and channels can be provided Channels with fidelity p are then modelled as:

$$\rho(p) = p \left|\phi_{00}\right\rangle \left\langle\phi_{00}\right| + \frac{1-p}{4}\mathbf{I}_4 \tag{1}$$

And gates of fidelity p_U are modelled as(here $|\psi_{perf}\rangle$ is the state after the gate is applied correctly):

$$\rho_U(p_U) = p_U \left|\psi_{perf}\right\rangle \left\langle\psi_{perf}\right| + \frac{1-p_U}{2}\mathbf{I}_2 \tag{2}$$

To estimate the fidelity and the success rate both protocols are simulated a number of times at each setting. The increments chosen for both the gate- and the channel fidelity are the interval from 0-1 with steps of 0.1. All possible combinations of these will be tested. These values were chosen as they provide insights as to how the protocols function ate a wide variety of fidelity's.

The number of runs per specific combination of gate-and channel fidelity is 100. This number of runs was chosen as it is feasible to run the simulation this often for every value, and it provides insights to the broad trends in success rate.

4 Results

In this section the results of the experiments will be presented.

4.1 Success rate

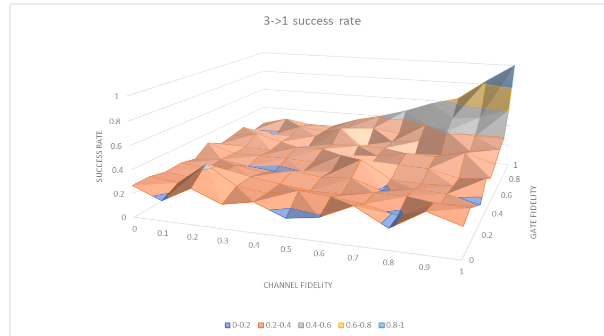

Figure 2: The $3 \rightarrow 1$ distillation protocol success rate data plotted

It can be seen from 2 that a high gate fidelity (≥ 0.8) is needed to achieve success rates higher than 0.4, just a high channel fidelity is not enough to reach above this threshold. The data showcases this by giving success rates for a channel fidelity of 1.0 which are not different for a higher or lower gate fidelity unless the gate fidelity goes above the previously described threshold of 0.8. The results also confirm the intuitive hypothesis that a combination of a high gate fidelity and a high channel fidelity is ideal for a higher success rate.

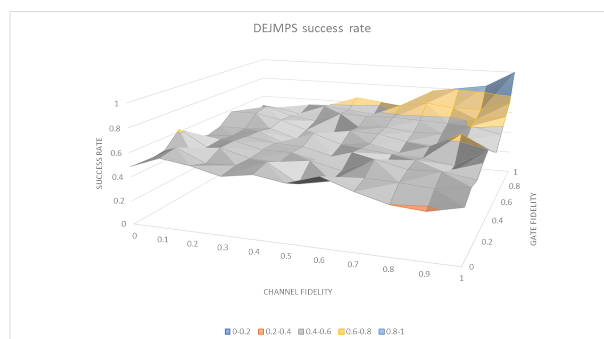

Figure 3: The DEJMPS distillation protocol success rate data plotted

Roughly the same holds up for the DEJMPS success rate. However, as can be seen in 3, the overall success rates are higher with similar combinations of gate and channel fidelity.

4.2 Fidelity

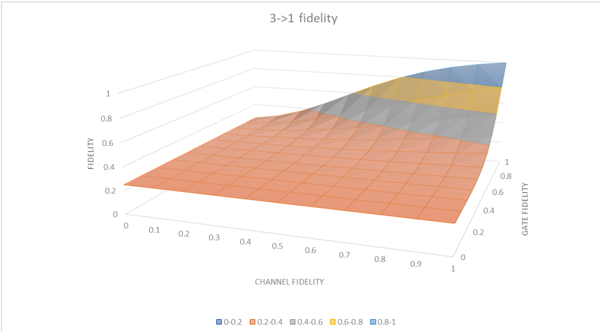

Figure 4: The 3 → 1 distillation protocol fidelity data plotted

A similar pattern emerges with the fidelity of the 3 → 1 protocol's fidelity compared to the success rate of the protocol: a high gate fidelity is needed to achieve a high fidelity overall, from there a higher channel fidelity allows the overall fidelity to scale upwards.

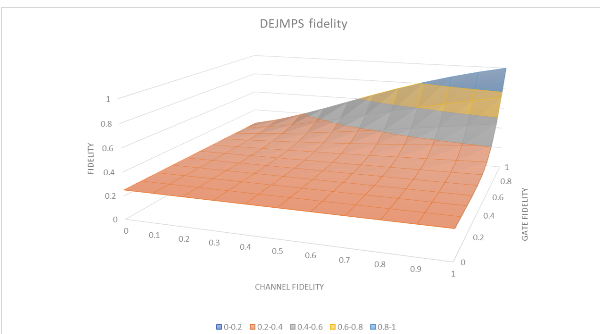

Figure 5: The DEJMPS distillation protocol fidelity data plotted

The curve is eerily similar to the one of the 3 → 1 protocol's fidelity. However, the curve is a slightly more flat around the $0.4 - 0.6$ mark of the channel fidelity compared to the 3 → 1 protocol's fidelity. Additionally the resulting fidelity is slightly lower than the fidelity achieved using the 3 → 1 protocol.

5 Discussion

The results that were achieved are not without their flaws. The amount of runs used to estimate the success rate, while enough to confidently speak on broad trends in the data, are not enough to provide accurate estimates on the actual success rate at specific parameters for the gate- and channel fidelity. Significantly increasing the number of runs to provide better estimates on the success rates is infeasible as it already took more than 24 hours to get this many runs for the 3 → 1 protocol.

6 Conclusion

As can be seen in the results the success rate of the 3 → 1 protocol are generally higher than the success rates for the DEJMPS protocol. Additionally, while the fidelity in the DEJMPS protocol is lower than the fidelity achieved with the 3 → 1 protocol, the difference is not incredibly high. Combined with the fact that for the 3 → 1 protocol 3 initial qubits are needed as opposed to just 2 for the DEJMPS protocol, one can imagine that when the cost for generating these initial qubits is non-trivial the DEJMPS protocol would probably be preferred. In other situations, where the generation of these initial qubits is not a limiting factor, and the fidelity needed on the achieved EPR-pair is of paramount importance, the 3 → 1 protocol would probably be better.

Furthermore, it can be clearly seen that the impact of the gate fidelity on the success rate of both protocols is significantly higher than the impact of the fidelity of the initial pairs. In addition to this, both protocols can be used to feed their resulting pairs to new rounds of the protocol thereby improving on the effects of the channel fidelity, and creating pairs of fidelity's that are arbitrarily close to maximally entangled (although this may require large amounts of rounds of the protocol). However, this only works when the fidelity of the resulting pair is higher than the fidelity of the initial pairs. In the results it can be seen that for these intervals this only occurs when the gate fidelity is larger than 0.9. Further research could look at defining the threshold for required gate fidelity for the protocols to function more precisely.

References

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A Appendix

A.1 3 → fidelity rate

gate_fidelity \channel_fidelity	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
0.1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.2501	0.2501
0.2	0.25	0.25	0.25	0.2501	0.2502	0.2503	0.2504	0.2505	0.2507	0.2509	0.2512
0.3	0.25	0.25	0.2502	0.2504	0.2508	0.2513	0.2519	0.2527	0.2536	0.2548	0.2561
0.4	0.25	0.2502	0.2506	0.2514	0.2525	0.2541	0.2561	0.2586	0.2617	0.2652	0.2694
0.5	0.25	0.2504	0.2516	0.2536	0.2565	0.2605	0.2656	0.2718	0.2793	0.2882	0.2984
0.6	0.25	0.2512	0.2539	0.2585	0.2651	0.2739	0.285	0.2985	0.3146	0.3332	0.3543
0.7	0.25	0.253	0.2593	0.2694	0.2835	0.3019	0.3245	0.3512	0.3818	0.4157	0.4526
0.8	0.25	0.2574	0.2718	0.2939	0.3236	0.3606	0.4037	0.4514	0.5019	0.5534	0.6042
0.9	0.25	0.2678	0.3005	0.3483	0.409	0.4783	0.5511	0.6225	0.6885	0.747	0.797
1	0.25	0.2908	0.3636	0.4631	0.5761	0.6875	0.7857	0.8652	0.9257	0.9695	1

A.2 DEJMPS fidelity rate

gate_fidelity \channel_fidelity	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
0.1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
0.2	0.25	0.25	0.2501	0.25001	0.2502	0.2502	0.2503	0.2503	0.2504	0.2504	0.2504
0.3	0.25	0.2501	0.2505	0.2507	0.2509	0.2512	0.2515	0.2517	0.252	0.2523	0.2526
0.4	0.25	0.2508	0.2516	0.2524	0.2533	0.2543	0.2553	0.2563	0.2574	0.2586	0.2598
0.5	0.25	0.2521	0.2543	0.2567	0.2594	0.2622	0.2651	0.2683	0.2717	0.2752	0.2788
0.6	0.25	0.2547	0.2601	0.266	0.2725	0.2796	0.2872	0.2954	0.3039	0.313	0.3224
0.7	0.25	0.2598	0.2713	0.2844	0.2989	0.3147	0.3318	0.3498	0.3687	0.3883	0.4084
0.8	0.25	0.2689	0.2917	0.3181	0.34745	0.3792	0.4128	0.4475	0.4829	0.5183	0.5534
0.9	0.25	0.2843	0.3270	0.3763	0.4304	0.4872	0.5449	0.6019	0.6569	0.7091	0.7579
1	0.25	0.3094	0.3846	0.47021	0.5603	0.65	0.7353	0.8138	0.8841	0.9461	1

A.3 DEJMPS success rate

gate_fidelity \channel_fidelity	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.48	0.58	0.54	0.48	0.52	0.48	0.56	0.47	0.4	0.38	0.44
0.1	0.47	0.6	0.49	0.5	0.55	0.4	0.45	0.5	0.49	0.47	0.51
0.2	0.49	0.47	0.56	0.49	0.48	0.52	0.46	0.58	0.52	0.53	0.49
0.3	0.62	0.48	0.56	0.5	0.5	0.44	0.55	0.43	0.53	0.44	0.56
0.4	0.49	0.49	0.54	0.5	0.49	0.43	0.52	0.49	0.48	0.44	0.6
0.5	0.45	0.47	0.53	0.43	0.55	0.47	0.5	0.44	0.52	0.65	0.52
0.6	0.48	0.48	0.5	0.42	0.45	0.54	0.53	0.48	0.46	0.54	0.45
0.7	0.58	0.58	0.53	0.45	0.39	0.52	0.54	0.51	0.52	0.61	0.55
0.8	0.44	0.6	0.52	0.49	0.58	0.57	0.58	0.57	0.56	0.62	0.66
0.9	0.44	0.54	0.49	0.59	0.57	0.54	0.65	0.69	0.66	0.66	0.74
1	0.5	0.44	0.54	0.55	0.65	0.62	0.58	0.77	0.81	0.85	1

A.4 3 → 1 success rate

gate_fidelity \channel_fidelity	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.27	0.17	0.3	0.2	0.25	0.15	0.19	0.31	0.17	0.33	0.25
0.1	0.27	0.2424	0.43	0.27	0.24	0.29	0.27	0.24	0.18	0.31	0.3
0.2	0.24	0.21	0.27	0.2727	0.24	0.21	0.21	0.25	0.25	0.22	0.23
0.3	0.25	0.23	0.28	0.24	0.28	0.25	0.27	0.19	0.23	0.22	0.16
0.4	0.22	0.24	0.2	0.18	0.2	0.23	0.36	0.21	0.2	0.21	0.3
0.5	0.31	0.23	0.23	0.23	0.22	0.31	0.23	0.27	0.26	0.2	0.23
0.6	0.24	0.23	0.22	0.27	0.28	0.16	0.26	0.31	0.3	0.36	0.28
0.7	0.26	0.3	0.24	0.18	0.26	0.27	0.27	0.13	0.25	0.31	0.35
0.8	0.29	0.22	0.24	0.26	0.36	0.16	0.27	0.3	0.42	0.39	0.39
0.9	0.20	0.18	0.28	0.25	0.31	0.33	0.29	0.4	0.49	0.52	0.61
1	0.22	0.29	0.25	0.25	0.34	0.4	0.47	0.57	0.64	0.83	1